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Adaptive Optics Control Strategies for Extremely Large Telescopes

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ABSTRACT

Adaptive optics for the 30-100 meter class telescopes now being considered will require an extension in almost every area of AO system component technology. In this paper, we present scaling laws and strawman error budgets for AO systems on extremely large telescopes (ELTs) and discuss the implications for component technology and computational architecture. In the component technology area, we discuss the advanced efforts being pursued at the NSF Center for Adaptive Optics (CfAO) in the development of large number of degrees of freedom deformable mirrors, wavefront sensors, and guidestar lasers. It is important to note that the scaling of present wavefront reconstructor algorithms will become computationally intractable for ELTs and will require the development of new algorithms and advanced numerical mathematics techniques. We present the computational issues and discuss the characteristics of new algorithmic approaches that show promise in scaling to ELT AO systems.

1. INTRODUCTION

Serious discussion is now taking place concerning the construction of 30, 50, even 100 meter optical telescopes. Certainly such monumental projects will require the most advanced technologies in structures and optical fabrication. Designing adaptive optics systems for telescope apertures of this size presents its own set of challenges. Some of these challenges are matters of cost and scale of components, i.e how do we build deformable mirrors, wavefront sensors, etc. with numbers of elements that scale with aperture area without incurring enormous costs or making the system physically too large. Additional challenges are in maintaining reliability and observing efficiency in the face of the greatly increased system complexity.

We can start the discussion with a broad-brush overview of the AO system concepts for extremely large telescopes (ELTs) and, from well-known scaling laws, determine the requirements for basic system components. We then consider the state of the art in these component areas and ask if rates of technology development will meet the ELT timelines.

2. AO CONCEPTS FOR ELT'S

A preliminary design of a 30-meter telescope is being pursued in a collaborative effort by the University of California and California Institute of Technology. The concept is a segmented mirror primary / monolithic secondary in a Ritchey-Chretien design, essentially a scaled-up version of the Keck 10-meter except that the segments are smaller to save on fabrication costs. Adaptive optics systems can be tailored to the wavelength band, instrument, and observing program. Here are four baseline systems:

1. Low-emissivity, low-order adaptive and active optics for imaging in the 5 - 20+ micron band.
2. Low-order, high sky coverage, single natural guide star system for infrared astronomy in the 2 – 5 micron band
3. Tomographic AO: multiple deformable mirrors at atmospheric conjugate layers, multiple guide stars for high sky coverage infrared Astronomy to 1 micron. This is good for extragalactic astronomy and probing dust obscured star forming regions.
4. "Extreme" AO: a high Strehl, narrow field adaptive optics system targeted toward probing the space close to bright stars in the near IR – this would be used in the direct detection of extra solar planetary systems, dust disks, etc.

We'll focus our attention in this paper mostly on systems 3 and 4, since they present the most interesting technological challenges.

Other large telescope projects may follow along similar baseline AO paths. At this stage in the development of the AO knowledge base however, we can start to hypothesize interesting "deployments" of AO systems that depart from the traditional. For example, one can envision building AO systems into each instrument package. If the components can be made small enough, the entire AO system could be put in an camera IR dewar, which reduces the thermal emissivity significantly. Infrared wavefront sensors are a distinct possibility in the near future as IR sensor arrays become faster and lower noise. There are two benefits to sensing in the IR: first natural stars tend to be brighter at near-infrared wavelengths, thus increasing the sky coverage, and second, there is some benefit in the wavefront error budget as Hartmann spots become diffraction-limited (see section 3).

We can also imagine incorporating the understanding of atmospheric physics and the understanding of the dynamics of the AO system into process of doing AO astronomy. There is an important ongoing effort to understand the nature and stability of the AO point spread function. Data from on-site seeing instrumentation and telemetry from the AO system itself will be combined to produce estimates of the point-spread function simultaneous with each observation. Perhaps with advances in the modeling, we will have the ability to *predict* seeing in advance, and so schedule AO observations accordingly.

3. ERROR BUDGETS AND SCALING LAWS

Simple strawman error budgets for ELT AO systems can be worked out for the sake of illustrating the component requirements. In order to produce reasonable Strehl images, say S=0.5, the total wavefront error must be kept to under about $\lambda/7.5$ rms. We use the Marechal approximation $S \sim \exp\{-\sigma_\phi^2\}$ (σ given in radians) to relate wavefront error to Strehl. This is about 130 nm at 1μ wavelength. Some of this error budget of course must be given over to the uncorrectable or unmeasureable parts of the optical train (non-common path errors). More of it must be given to wavefront aberrations caused by the telescope optics, for example, segment phasing, tilt, and placement, which, although partially corrected by adaptive optics, introduce spatial or temporal frequency components that are not completely corrected. Finally we are left with what the atmosphere gives us. For this there are well-known scaling laws. Table 1 gives representative error budgets for 30-meter and 100-meter apertures. They include three terms: DM fitting error, i.e the limit due to finite spatial frequency of the wavefront sensor/corrector, measurement error, which is the limit due to noise in the guidestar detection process, and servo error which is the limit due to finite temporal frequency tracking of the wavefront by the closed loop AO system. We neglect for now the complicating additional terms such as cone-effect with the laser guidestars, anisoplanatism, and the like, since they depend in a more complex manner on the particulars of the observation, imaging field, and multi-conjugate AO geometry. Suffice it to say that the cone effect error would dominate in single laser guidestar AO system on a 30 meter telescope. The multi-guidestar tomographic reconstruction is required to bring the cone-effect term down to size, but at the expense of introducing other small terms that depend in complicated manner on the field angle, brightness and position of auxiliary natural guidestars.

D=30 m		WF error	D=100 m		WF error
Atmospheric fitting	n_{DOF} =10,000	64	Atmospheric fitting	n_{DOF} =90,000	64
WF sensor noise	$m_V = 10$	60	WF sensor noise	$m_V = 10$	69
Servo bandwidth	$f_s = 335$ Hz	73	Servo bandwidth	$f_s = 335$ Hz	73
RSS		114 nm	RSS		119 nm

Table 1. Representative error budgets for AO systems on ELTs with an 8'th magnitude on-axis natural guidestar

Scaling laws for the rms wavefront error contribution of each of these terms is

Atmosphere fitting	~	$(d/r_0)^{5/6}$
WF sensor noise	~	$\ln(\mathrm{DOF})\, b^{-1/2} f_s^{1/2} d^p r_0^{(1-p)}$
Servo bandwidth	~	$f_s^{-5/6}$

Where d is the DM actuator spacing, r_0 is Fried's seeing parameter (~50 cm at 1 micron at a good astronomical site), DOF is the number of degrees of freedom on the DM, b is the brightness of the guidestar ($\sim 10^{m_V/2.5}$), and f_s is the wavefront measurement sampling rate. p is an exponent in the wavefront measurement error term which depends on r_0 at the wavelength of the wavefront sensing. For $d \gg r_0$, where the guidestar size is dominated by seeing, $p = 0$. In this regime, an increase in subaperture size increases the number of photons but simultaneously degrades the spatial sampling of the wavefront (areas of the aperture effected by individual slope measurement errors are larger). For $d \ll r_0$ where the guidestar size is dominated by diffraction, the exponent approaches $p = -1$. In this regime the diffraction-limited guidestar spot gets smaller with subaperture size, improving the centroiding. Present day AO systems use visible light and subapertures are typically larger than r_0, a regime where there is no first order benefit to increasing subaperture size. Future AO systems may likely use infrared wavefront sensors and larger subaperture sizes would provide a benefit to the error budget that trades against the atmospheric fitting term.

We can now write down the "technology scaling laws" that relate difficult cost or difficulty factors to telescope size. Certainly, to keep the atmospheric fitting error fixed, the number of deformable mirror degrees of freedom must increase with telescope diameter squared, and along with it the size of the wavefront sensor array

$$DOF \sim D^2$$

$$n_{pixels} \sim D^2$$

Secondly, to keep up the wavefront sensor read rate, the pixel readout speed must increase correspondingly

$$f_{pixel} = f_s D^2$$

Finally, if we use the standard least-squares reconstructor algorithm, where we need to calculate commands for each of the degrees of freedom given each of the wavefront sensor slopes with a full matrix-muliply, the computation rate scales with

$$FLOPS \sim f_s D^4$$

4. COMPONENT TECHNOLOGIES

4.1 Deformable mirrors

The largest "conventional" deformable mirror (one using piezo-transducers) had on the order of 2000 actuators. Buttable monolithic arrays of transducers are under development by Xinetics, inc. that may exceed this number, but still scale in cost on the order of $1k per actuator as has been the case for a number of years with these types of mirrors. Two ~900-actuator mirrors are in use today (on the AMOS 3-m telescope on Haleakela and the 3-m at Starfire optical range). The Keck AO systems use 349 actuator mirrors, as does the Mt Palomar 5-m telescope AO system and the Mt Wilson 2.5-m.

MEMs (micro electro-mechanical device) technology is a promising wave of the future, both in terms of cost per actuator and scalability to very large numbers of degrees of freedom. There small size is also attractive from the standpoint of overall AO system size, but too small presents a problem of AO system field of view. This is because the beam demagnification from ELT aperture (30-100 m) to MEMs (~10mm, say) would lead to and extreme magnification of field angle. Limiting incidence angles on the MEM to 45 degrees say would only result in an arcminute field on the 30 m and 20 arcsec on the 100 m. The push is to make 100 and 200 mm size MEM devices for astronomy. A number of such devices technologies are under development in a collaboration lead by the Center for Adaptive Optics. A 144 degree-of-freedom, 3 x 3 mm device produced by Boston Micromachines has recently been tested in our laboratory. A 1000 actuator device is due to be delivered later this year. The cost of these devices is roughly $20 per degree of freedom.

The requirement for multi-conjugate adaptive optics (MCAO) on the 30-meter telescope is 10,000 degrees of freedom. Extreme AO, with 10 cm subapertures on the 30-meter, will need upwards of 100,000 degrees of freedom. This would be possible on a 200 mm MEMs device with 0.6 mm actuator spacing, which is a reasonable extrapolation for this technology over the next 10 years or so. A 100-meter telescope needs this size device for routine MCAO (scaled to 30 cm subapertures).

4.2 Wavefront sensors

Wavefront sensors present another dimension of challenge for astronomical adaptive optics. In recent years the push has been toward larger, faster, and lower noise CCD arrays for visible light wavefront sensing. The 80 x 80 CCD-39 by Marconi Applied Technologies (formerly EEV) has 3 electrons read noise and is a choice for the Gemini MCAO system. MIT/Lincoln Labs has developed 3 electron 64 x 64 (used at Keck and Lick) and 128 x 128 (used at Starfire Optical Range) CCDs that read as fast as 1500 frames per second.

At this point, lower noise may not necessarily be the goal; the photon noise from the guidestar will dominate assuming on the order of 100 detected photo-events per subaperture per frame will be necessary for reasonable atmospheric correction. The big issue (for big telescopes) is size and readout speed. The 128 x 128 CCDs with 2 x 2 pixels per subaperture, and no pixel guard band, can accommodate up to 4000 subapertures. 30-meter MCAO will need 10,000, so this requires at least a 200 x 200 array. Furthermore this array must read out at the standard, say 500 Hz, frame rate for correction in the IR. Depending on the parallel output-port architecture, this may not require increased pixel read rate over existing devices.

The Extreme AO concept for the 30 meter telescope will require a much larger sensor, on the order of 600 x 600 pixels. Also, for Extreme AO to achieve high Strehl commensurate with sensing on d = 10 cm subapertures, the sample rate must be higher by a factor of ~3 than the d = 30 cm MCAO sensor. That makes 1.5 kHz frame rate the baseline for Extreme AO. Since the 64 x 64 and 128 x 128 Lincoln Lab chips have 4 and 16 readout ports respectively to achieve their high frame rates, let us say we want to scale the number of amplifiers to achieve similar frame and pixel read rates with the 600 x 600 array. The number of readout ports required is around 300. Furthermore, the frame transfer latency, which is ~15μs for the 64 x 64 chip, scales to ~70μs for the 600 x 600 case, which is about 10% of the frame integration time. The frame transfer latency manifests itself as smearing of guidestar light along the direction of frame transfer and may become a significant contributor to the wavefront sensor error budget.

4.3 Lasers

The current working technology for sodium beacons at the Lick and Keck observatories is a high-powered tunable dye laser adjusted to the 589 nm sodium wavelength and pumped by doubled Nd/YAG lasers (532 nm). Up to 2 kW output power has been demonstrated with this technology. This type of laser emits a short pulse (100 ns) which saturates the Sodium layer at a pulse energy of ~0.3 mJ/cm^2. To keep below the saturation level, the laser pulse repetition frequency is chosen to keep the peak power low but maintain high average power. The Lick laser runs at 11kHz so, for a 1 arcsecond beacon (0.5 m at the sodium layer), ~30 Watts average power is needed to saturate. The Keck laser runs at 25 kHz because the goal there is to produce a 0.6 arcsecond (0.3 m) spot.

"Solid state" lasers, that is, lasers with crystaline instead of liquid lasing material, are arguably easier to deal with and operate in the observatory environment. The difficulty is in producing high power at the sodium wavelength. There are several varieties currently under development but all have a foreseeable power limit of ~10 Watts.

How much power do we need? To first order, the single-laser power requirement scales with subaperture size d^2 and wavefront sampling frequency f_s. This would imply that for correction in the near IR (1-2μ), since d and f_s are the same on the 30 (or 100) meter as on the 10 meter, ~10 W would be sufficient. Unfortunately, as the telescope diameter gets larger the average distance of the laser beam launch aperture from any subaperture get larger, and this results in an apparent elongation of beacon. Since the wavefront measurement error is directly proportional to beacon size, this elongation drives a higher laser power requirement. If for example, the laser is elongated by a factor of 5 (as it is for the 30 meter on an edge subaperture if the laser is projected from behind the secondary), then 25 times as much laser power is needed to recover the accuracy in centroiding. There are methods that have been proposed for compensating for the elongation, which we discuss in section 6.

4.4 Computers

The traditional least-squares matrix-vector-multiply reconstructor has the form:

$$s = Ha$$

$$\hat{a} = \left(H^T H\right)^{-1} H\, s$$

where $\boldsymbol{s}$ is the vector of wavefront sensor slopes, $\boldsymbol{a}$ is the vector of DM actuator commands, $\boldsymbol{n}$ is the wavefront sensor noise, and $\boldsymbol{H}$ is the interaction matrix, which is of dimension (number of actuators) × (number of sensors). Weighted least squares, Optimal (A++), and Modal (if all modes are retained) reconstructors all have similar computational structure to the least-squares formulation above, which requires $O(n_{DOF}^2)$ operations to perform the matrix multiplication. Keep in mind that for multi-conjugate AO, all the sensor readings are strung into the vector $\boldsymbol{s}$ and all the DM actuator values are strung into the vector $\boldsymbol{a}$. That is, in general, the MCAO reconstructor requires a "full" matrix multiplication.

We'll compare to the Keck AO reconstructor handling 349 degrees of freedom which runs on a set of 16 Intel I-860 high-speed floating point processors. A reconstruction requires on the order of 1.2×10^5 calculations per 1.5 ms sample period. For the 30-meter with a single guidestar and 10^4 degrees of freedom, the calculations are 10^8 / 1.5 ms. For MCAO on the 30 meter with, say, 5 conjugate DMs and 5 guidestars, the calculations are 2.5×10^9. For Extreme AO on the 30 meter, or MCAO on the 100 meter, with on the order of 10^5 degrees of freedom, the calculation requirements are phenomenal, ~ 10^{10}. That's 10^5 Keck processors!

Even with the fast-paced growth of processor power, it's hard to imaging a scaling of computational power to this degree. We explore this more in the next section.

5. ALGORITHMS FOR WAVEFRONT RECONSTRUCTION

5.1 Dealing with Moore's law

In 1965, Intel CEO Gordon Moore observed that processor capability doubles every 18 months, a ``law" that has proven accurate up to today. If we use this law to predict when we will have the technology to meet ELT computational needs (a 10^5 improvement) we find it will take 25 years technology growth. Let's say we use a factor of 100 more processors, then the 10^3 improvement takes 15 years.

If we are to use wavefront slope sensors (and there is good reason to, see subsection 5.2) clearly we need to develop fast reconstructor algorithms, i.e. algorithms that scale slower than $O(n_{DOF}^2) \sim O(D^4)$\$. One approach is to use sparse matrix techniques, taking advantage of the fact that $\boldsymbol{H^T H}$ is mostly composed of zero, or near-zero elements. It also is highly structured with nearly repeated sub-elements. To lead along this direction, we point out that there is a differential equation analogy to the wavefront reconstruction process that essentially involves solving Poisson's equation:

$$\vec{s} = \nabla \varphi$$
$$\nabla^2 \hat{\varphi} = \nabla \cdot \vec{s}$$

A differential equation can be rewritten in terms of an integral equation, where the ``kernel" is known as the Green's function. In fact, the Green's function for Poisson's equation is quite well-known (electrostatics), but it does not help computationally since the extent of that function is effectively global. If however a Green's function can be found that has local influence, say, starting not from $\nabla \bullet s$ but s directly, then there is hope for an $O(n_{DOF}) \times$ (local influence extent) algorithm.

Visible struture in the matrix $\boldsymbol{(H^T H)^{-1} H^T}$ suggests that at least approximately there may a Green's function having local influence. However, a workable fast algorithm is yet to be worked out.

Another type of fast algorithm that is here today is one based on Fourier Transforms. The diffrential analogy is:

$$\vec{S} = i\vec{k}\Phi + N$$
$$\hat{\Phi} = \frac{-i\vec{k} \cdot \vec{s}}{k^2}$$

Using the fast-fourier-transform (FFT) this equation can be solved in $O(n_{DOF}) \log(n_{DOF})$ operations. For 30-meter Extreme AO and 100-meter MCAO, this brings the calculation requirements down to a more manageable $\sim 10^6$ calculations / ms. The FFT is an approximate method however, since the assumption is that the slope data are on a square (or rectangular) region, while telescope apertures tend to be roughly circular. The data outside the aperture data is, erroneously, set to zero in this algorithm which results in wavefront reconstruction errors, particularly near the edges of the aperture. This algorithm is the subject of ongoing research, including laboratory experiments at LLNL.

5.2 The noise propagator

The noise propagator factor quantifies how much the noise in the wavefront sensor is amplified by the wavefront reconstuctor. There appears to be a fundamental law, based on Poisson equation inversion in the Fourier domain, that says that noise propagation factor scales with $\ln(n_{DOF})$ for slope reconstructors[1]:

$$\tilde{\Phi} = \frac{i\vec{k} \cdot \vec{N}}{k^2}$$
$$\sigma_{\tilde{\varphi}}^2 = \sigma_n^2 d^2 \underbrace{\pi^{-2} \ln(n_{DOF}/4)}_{\text{Noise Propagator}}$$

Noise-optimal algorithms, such as those in Walter Wild's A++ codes[2], reduce the coefficient, however the $\ln(n_{DOF})$ scaling remains. The noise propagator for Keck is 0.45. For 30-meter MCAO it is 0.79, and for 30-meter Extreme AO (100-meter MCAO) it is 1.03. The makes the rms noise in 100-m AO $(1.03/0.45)^{1/2} = 1.5$ times worse, given the same brightness guidestar.

Curvature sensors are even worse: the noise propagator scales with n_{DOF}. Wavefront sensor rms noise would be $(10^5/349)^{1/2} \approx$ 17 times worse on a 100-meter than on the 10-meter.

These factors are important because they translate into additional laser power needed to recover wavefront sensor accuracy by increasing the signal-to-noise ratio.

6. LASER GUIDESTARS' SPECIAL PROBLEMS

Using laser beacons on large telescopes presents special problems. On 10-meter and smaller telescopes only a single laser guidestar is needed to correct the wavefront at IR wavelengths. Beyond that size, cone effect (also known as focal anisoplanatism) becomes significant, stemming from the fact that the cone of light coming down from the finite altitude laser guidestar does not fully sample the atmospheric volume through which the starlight passes. Multiple laser beacons appropriately placed on the sky will provide enough probe paths for tomographic reconstruction and effectively eliminate cone effect.

Now the question is from where does one project the lasers? As pointed out earlier, no matter where the laser projector is positioned, the Hartmann spot will appear elongated in some subapertures. Andreas Quirrenbach has pointed out that the elongation effect is really caused by beacon light coming from different focus distances as a function of vertical position in the sodium layer. This shows up as a radial pattern on a wavefront sensor, i.e. a composite of many foci. A possible solution to this issue is to dynamically focus-track laser pulses as they pass through the sodium layer. Such a tracker might resemble the membrane mirror like those used in curvature wavefront sensors. But quick calculations (sag = $\lambda/4 \times \Delta z$/(depth of focus)) show that in order to track the 10 km focus change, a membrane mirror must move ~68 μm at a rate of 30 kHz. This quite a bit larger than present membrane mirrors can move (the largest being about 10 μm). Also, in contrast to the curvature membrane mirrors, surface figure of the mirror is critical, i.e. it must introduce only focus, not spherical or other modes, or this will be a source of uncalibrated wavefront sensor error.

Another solution was suggested by Jerry Nelson. We project lasers from two separated launch locations but to the same point in the sodium layer so that the elongated beacons form cross patterns on the Hartmann wavefront sensor detector. The centroid across only the thin dimension of the elongated guide star is just as accurate as that of a non-elongated guide star. We use the second guidestar to centroid along a second direction (its thin dimension), thus forming the second component of wavefront slope, and, combined with the first measurement, this provides the wavefront gradient. The lasers are pulsed so measurements of each are taken separately. Of course, this technique uses twice as much power to reproduce the accuracy of a non-elongated guidestar, but this is much better than using 25 times the power to recover from 5 times elongation.

7. CONCLUSION

In this paper we have discussed the technology issues that are along the critical path to fielding adaptive optics on extremely large telescopes. The potential benefit to astronomy of a 30 to 100 meter class, diffraction-limited telescope is almost beyond imagining and presents an exciting prospect for the future. Many of the components required however are not simple extensions to present day technology but require radically new approaches to make them feasible and to keep down cost.

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